WAVE RIDER CONFIGURATIONS BEHIND PARABOLIC SHOCK WAVES

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Exact solutions exist to inverse gas-dynamic problems involving supersonic flows and planar shock waves [1-3], and there are also solutions for conical shock waves [4-8]. The bodies may be better than those equivalent in volume and length constructed from the stream lines behind the planar shock waves not only as regards load-carrying characteristics (for Λ wings) but also as regards resistance (in the case of stars). Naturally, a further development is to derive optimal bodies based on curvilinear shock waves. A method is proposed for extending gas-dynamic design principles by means of surfaces with variable curvature [9] and bodies of regenerative shape [10]. A comparison has been made between wave riders having Λ-shaped cross sections and other bearing bodies formed by flow surfaces behind planar or axisymmetric shock waves [11].

Here we present numerical solutions for the flow around wave riders based on parabolic shock waves.

Calculation Method. Let the shape be known for an arbitrary attached shock wave (SW), where it is necessary to calculate the flow parameters behind it. We write the basic planar-flow equations in a rectangular coordinate system (the x axis coincides with the velocity vector for the unperturbed flow, and the y axis is perpendicular to the x axis and directed downwards):

$$\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}=0,\quad \frac{\partial v_x}{\partial x}v_x+\frac{\partial v_x}{\partial y}v_y+\frac{k}{\rho}\frac{\partial p}{\partial x}=0,\quad \frac{\partial v_y}{\partial x}v_x+\frac{\partial v_y}{\partial y}v_y+\frac{k}{\rho}\frac{\partial p}{\partial y}=0,$$
$$v_x=\frac{v_{x0}}{v_\infty},\quad v_y=\frac{v_{y0}}{v_\infty},\quad p=\frac{p_0}{p_\infty},\quad \rho=\frac{\rho_0}{\rho_\infty},\quad k=\frac{p_\infty}{\rho_\infty v_\infty^2}=\frac{1}{\varkappa M_\infty^2}. \tag{1}$$

Here v_{x0} and v_{y0} are the components of the velocity vector along the x and y axes, p_0 pressure, ρ_0 density, $\varkappa$ the specific-heat ratio, and v_∞, p_∞, ρ_∞ the unperturbed-flow parameters.

We supplement (1) with a condition for constant entropy along the stream lines:

$$\frac{\partial}{\partial x}\left(\frac{p}{\rho^\varkappa}\right)\frac{v_x}{v}+\frac{\partial}{\partial y}\left(\frac{p}{\rho^\varkappa}\right)\frac{v_y}{v}=0,\quad v=\sqrt{v_x^2+v_y^2}. \tag{2}$$

We supplement (1) and (2) with

$$\frac{\partial g_m}{\partial x}\cos(l,x)+\frac{\partial g_m}{\partial y}\cos(l,y)=\frac{\partial g_m}{\partial l}, \tag{3}$$

where $\partial g_m/\partial l$ ($m=1,\ldots,4$, $g_1=v_x$, $g_2=v_y$, $g_3=p$, $g_4=\rho$) are derivatives along a certain direction at the SW, while $\cos(\ell, x)$ and $\cos(\ell, y)$ are the direction cosines for that direction.

We thus have a closed system containing eight linear algebraic equations in (1)-(3) in terms of eight unknown first derivatives for the flow parameters at the SW $\partial g_m/\partial x$, $\partial g_m/\partial y$.

We determine the unknown parameters behind the SW numerically from the values at it. We establish the range in which the body, at present unknown, affects the induced SW. We take this range as one and divide it up into N small equal parts Δx_0 (Fig. 1: 1 SW, 2 characteristic, 3 stream line j = const, 4 i = const, 5 bottom section plane). The coordinates of the working points are x_{ij} and y_{ij} (the points where the flow parameters are calculated), in

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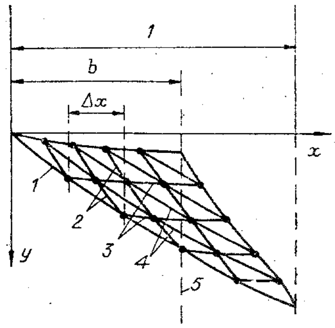

Fig. 1

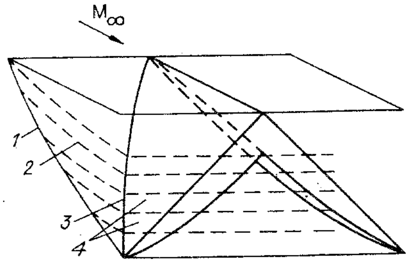

Fig. 2

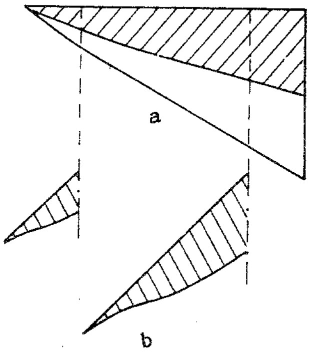

Fig. 3

which the subscripts i = 1, $1 \leqslant j \leqslant N + 1$ correspond to the calculation points on the SW ($x_{1j} = x_{1j-1} + \Delta x_0$, $y_{1j} = f(x_{1j})$, $y = f(x)$ is the SW equation).

We denote the unknown flow parameters and their first derivatives at the working points by v_{xij}, v_{yij}, p_{ij}, ρ_{ij}, $(\partial v_x/\partial x)_{ij} \ldots (\partial\rho/\partial y)_{ij}$. The derivatives of those parameters with respect to the SW direction are calculated from $(\partial g_m/\partial l)_{ij} = (g_{mij+1} - g_{mij})/R_{ij}$, $R_{ij} = \sqrt{\Delta x_0^2 + (y_{ij+1} - y_{ij})^2}$, and the direction cosines from $\cos(l, x)_{ij} = \Delta x_0/R_{ij}$, $\cos(l, y)_{ij} = (y_{ij+1} - y_{ij})/R_{ij}$.

At the first step (for i = 1), we derive the unknown first derivatives of the flow parameters at the SW from the algebraic equations. The second derivatives are calculated by differentiating each equation in (1)-(3) with respect to x and y. Then one obtains a system of 16 linear algebraic equations in the 12 unknown second derivatives $(\partial^2 v_x/\partial x^2, \partial^2 v_x/\partial x \partial y \ldots \partial^2\rho/\partial y^2)$ and four unknown first derivatives of the direction cosines: $\partial \cos(l, x)/\partial x$, $\partial \cos(l, x)/\partial y$, $\partial \cos(l, y)/\partial x$, $\partial \cos(l, y)/\partial y$. The first derivatives of the flow parameters are taken in this system from the solution to the system in terms of the first derivatives.

In that way, one can calculate derivatives of any order, and then Taylor's formula

$$g_{mi+1j} = g_{mij} + \left(\frac{\partial g_m}{\partial x}\right)_{ij} \Delta x_{ij} + \ldots + \left(\frac{\partial^n g_m}{\partial y^n}\right)_{ij} \Delta y_{ij}^n \tag{4}$$

is used to derive the unknown flow parameters in the second step behind the SW i + 1 = 2(1 ≤ j ≤ N). One similarly calculates the flow parameters at the next step (i + 1 = 3, 1 ≤ j ≤ N − 1) from the known ones on the line i = 2 and so on up to i = N, j = 1 (Fig. 1).

The stream lines are taken as rectilinear at each step within the flow behind the SW (i > 1) and are aligned to the velocity vector at the previous step. The increments Δx_{ij} and Δy_{ij} in (4) along the x and y axes correspondingly are taken in order to ensure a stable solution such that the characteristic in the first family with respect to the velocity v_{ij} emerging from point i + 1j passes through the point ij + 1 (Fig. 1).

The coordinates at point ij are denoted by x_1 and y_1, and those at ij + 1 by x_2 and y_2, and those at i + 1j by x_3 and y_3. The direction cosines for the straight line joining points 3 and 1 are $l_1 = (x_3 - x_1)/E_1$, $m_1 = (y_3 - y_1)/E_1$, $E_1 = [(x_3 - x_1)^2 + (y_3 - y_1)^2]^{0.5}$, and those for points 2 and 3 are $l_2 = (x_2 - x_3)/E_2$, $m_2 = (y_2 - y_3)/E_2$, $E_2 = [(x_2 - x_3)^2 + (y_2 - y_3)^2]^{0.5}$ and then the stability condition can be written as

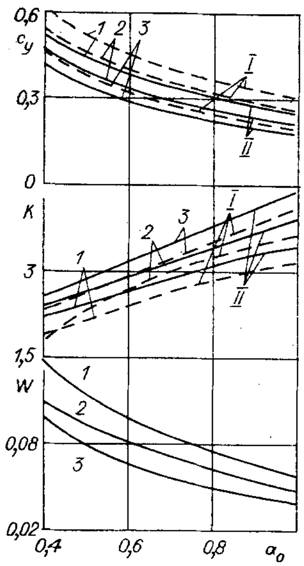

Fig. 4

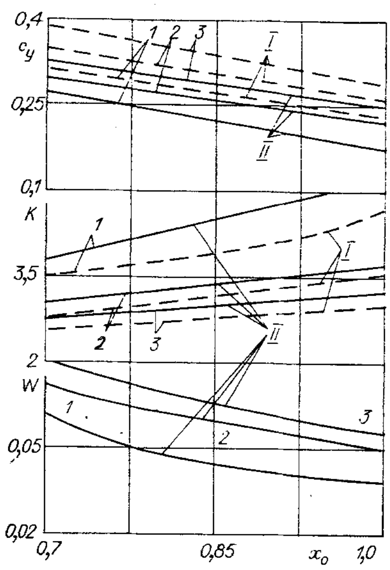

Fig. 5

$$l_1 l_2 + m_1 m_2 = \cos \mu_{ij}$$

$$(\sin \mu_{ij} = a_{*ij}/v_{ij} M_\infty, \quad a_{*ij} = \sqrt{p_{ij}/\rho_{ij}}, \quad v_{ij} = [v_{xij}^2 + v_{yij}^2]^{0.5}).$$

The obvious relations $x_{i+1j} = x_{ij} + \Delta x_{ij}$, $y_{i+1j} = y_{ij} + \Delta y_{ij}$, $\Delta y_{ij} = v_{yij}/v_{xij} \Delta x_{ij}$ have been used in solving the system by iteration on Δx_{ij}.

Figure 2 shows the body: 1 shock wave, 2 stream line, 3 leading edge, 4 carrying body.

Results. A computer program was written and bodies were constructed having parabolic shock waves, equation $x = a_0 y^2$ (wing II), and the aerodynamic characteristics were calculated. These were compared for identical values for the volume W, length b, and angle γ. Friction forces were neglected in calculating the frontal resistance, while the bottom pressure was taken as approximately equal to that in the unperturbed flow.

The calculations were performed with N = 70 up to second-order derivatives. For that N, the flow parameters calculated up to the first derivatives differed from the analogous ones calculated up to the second by less than 2.5%. Calculations on the same parameters with N = 40 differed from the latter (N = 70) by less than 3%, while those with N = 25 differed from those with N = 40 by 8%. We also calculated the entropy and total enthalpy behind the parabolic shock waves. The relative errors in these parameters along the various stream lines were hundredths of a percent.

Figure 3 shows the body sections (wings II) with taper angles (angles between the upper surfaces) $\gamma = 90°$ with a symmetry plane (a) and with planes $x = x_0 + 0.5b$ and $x = x_0 + b$ (bottom section) (b), where x_0 is the coordinate along the x axis for the point at which the SW is attached to the leading edge. The longitudinal section is slightly convex on the incident side, while the transverse section is slightly wavy.

Figure 4 shows how the lift coefficient c_y, aerodynamic quality factor K, and volume W (for wings I and II) vary with a_0, which characterizes the shape of a parabolic shock wave for various x_0 [M = 4; 1) x_0 = 0.7; 2) 0.85; 3) 1]. As a_0 increases, the lift coefficient and the volume decrease, while the aerodynamic quality factor improves because the inclination of the shock wave and the body thickness decrease as a_0 increases. As x_0 increases, the $c_y = c_y(a_0)$, $W = W(a_0)$ curves fall, while the aerodynamic quality factor improves, since a parabolic shock wave shows decreased inclination as x_0 increases.

Calculations were also performed on the analogous characteristics as functions of x_0 [Fig. 5: a_0 = 0.8; 1) M = 3; 2) 4; 3) 5], which showed that the wings II had aerodynamic quality better than I throughout the range in x_0. As M increased, so did the lift coefficient and volume, while the aerodynamic quality factor fell, since a given SW shape caused the thickness to increase with M. One obtains a distinct shape for each of the various M, a_0, and x_0. The calculations also showed that the volume increases with γ, while the lift factor for the II wings and the aerodynamic quality factor remain virtually unchanged.

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